A Novel Approach for Time Domain Analysis of Transient Currents in Conductors

T. A. G. de Tolosa, J. M. Janiszewski, and E. V. dos S. Pouzada
Mauá School of Engineering - Mauá Institute of Technology.
Praça Mauá, 1 CEP 09580-900 São Caetano do Sul – SP – Brazil
thiago@maua.br

Abstract — Time domain solutions for electromagnetic fields in conductive media, considering complex structures, can generally be obtained only by applying numerical procedures. Aiming at the analysis of transient currents, the paper shows the application of the Moment Method in association with a finite difference approximation of time derivatives which leads to a linear system characterized by matrix-form equations. Thus, currents are determined at each chosen time step in some specific points of the spatial subdivision of the geometry being considered.

When solutions are desired for transient problems involving non-homogeneous media, the static Finite Element Method can be applied in order to determine the coefficients of the problem main matrix.

Examples of calculated transient current distributions in some shielding configurations are presented.

I. Introduction

Time varying electromagnetic fields in conductive media can be analyzed in terms of the current density vector, usually defined by an integral equation, whose analytical solution is possible only in simple configurations [1]. For arbitrary complex structures, numerical procedures are employed and, among them, the Moment Method (MoM) can be applied [2]. In the case of transient phenomena, a time domain analysis is indicated to obtain accurate solutions, and then the Moment Method is employed in conjunction with a finite difference approximation of the time derivative, leading to matrix-form equations [3,4].

As shown in [5], the application of the proposed methodology is conditioned to the use of appropriate time and space subdivisions for the procedure to be stable.

When dealing with non-homogeneous media, the elements of the system main matrix are obtained by means of a static Finite Element Method (FEM) version available in the MATLAB PDE toolbox.

The paper shows application examples of the procedure considering the shielding effect [6] of magnetic and non-magnetic bodies in different situations concerning transient currents.

II. Fields in Conductors

Current density $\vec{J}(t)$ in a conductive medium with conductivity σ, excited by a given electric field $E_o(t)$, can be calculated by:

$$\vec{J}(t) = -\sigma \cdot \frac{\partial \vec{A}(t)}{\partial t} + \vec{J}_o(t) \qquad (1).$$

$\vec{A}(t)$ is the magnetic vector potential, defined by an integral of $\vec{J}(t)$ and $\vec{J}_o(t) = \sigma \cdot E_o(t)$.

Considering a bi-dimensional problem related to a very long non-magnetic conductor that is subdivided into filamentary elements, (1) can be written as the integral equation:

$$J(x,y,t) = \frac{\mu_o \cdot \sigma}{2\pi} \cdot \frac{\partial}{\partial t}\left[\int_S J \cdot \ln(\rho)\, dS \right] + J_o(x,y,t) \qquad (2).$$

S is the cross section area of the conductor, μ_o is the permeability of free space, and ρ is the distance between an element of the conductor (with an elementary current density J) and the (x,y) position of the point under analysis.

A discrete form of (2) can be written applying the Moment Method and substituting the time derivative by a finite difference approximation. Choosing a backward approximation, the following equation results:

$$[I + C] \cdot [J]^{(n+1)} = [C] \cdot [J]^{(n)} + [J_o]^{(n+1)} \qquad (3).$$

The upper index n is associated to $t = n \cdot \Delta t$ (Δt is the computational time step), $[I]$ is the identity matrix and $[C]$ is a matrix whose general element c_{ki} is a coefficient that gives the contribution of element i on element k of the spatial subdivision. Assuming filamentary elements, with square cross section of area $\Delta S = d^2$, for $k \neq i$,

$$c_{ki} = -\frac{\mu_o \cdot \sigma}{2\pi \cdot \Delta t} \cdot \ln\left(\sqrt{(x_k - x_i)^2 + (y_k - y_i)^2} \right) \cdot \Delta S \qquad (4)$$

and for $k = i$,

$$c_{ii} = -\frac{\mu_o \cdot \sigma}{2\pi \cdot \Delta t} \cdot \ln(0.44705 \cdot d) \cdot \Delta S \qquad (5).$$

In situations involving non-homogeneous regions (presence of magnetic materials where $\mu \neq \mu_o$ for example), the static FEM can be used in order to obtain the magnetic vector potential A_z in some pre-determined points of the conductors (considering filamentary currents imposed). The numerical relationship obtained between the potential and the imposed currents permits, in this case, to determine matrix $[C]$.

It is worth to note that as the proposed method works in a time step by step scheme, the solution for transient electromagnetic fields problems is not affected by Gibbs phenomenon, as in the case of frequency domain solutions.

The procedure was validated considering various situations with known solution and proved to be an efficient tool for analyzing problems of practical interest.

III. Application

Fig. 1 shows a cross section of a magnetic shielding system for two source conductors (c_1 and c_2) and a victim conductor c_3. Both conductor c_3 and the shielding are supposed to have their extremities short circuited ($E_o(t)=0$). Conductors c_1 and c_2 are subjected to imposed fields $E_o(t)$ so that the associated currents J_{c1} and J_{c2} are as indicated in Fig. 1. The system was configured with 64 filamentary elements (52 dealing with the shielding and 12 with the conductors) leading to a matrix [C] with 64x64 coefficients. The time step adopted was $\Delta t = 1\ \mu s$.

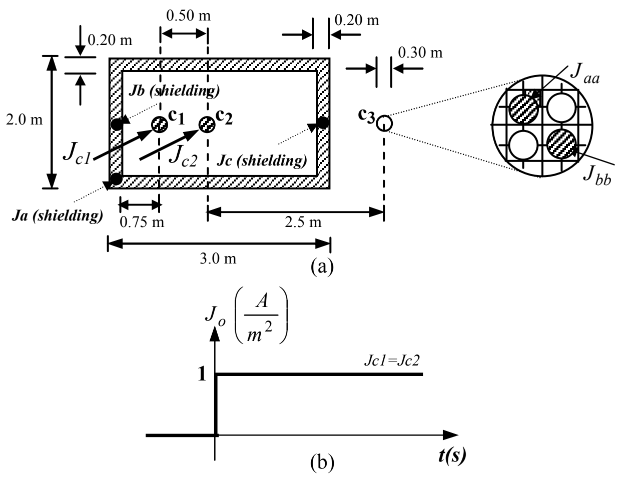

Fig. 1. (a) configuration of the shielding problem; (b) excitation of source conductors.

The parameters of the conductors used in this simulation were σ = 5.8·10^2 (S/m) and $\mu = \mu_o$. The shielding was considered, in this case, with σ = 5.8·10^2 (S/m) and $\mu = \mu_o$.

Fig. 2 shows the waveforms of current densities at different points of the system.

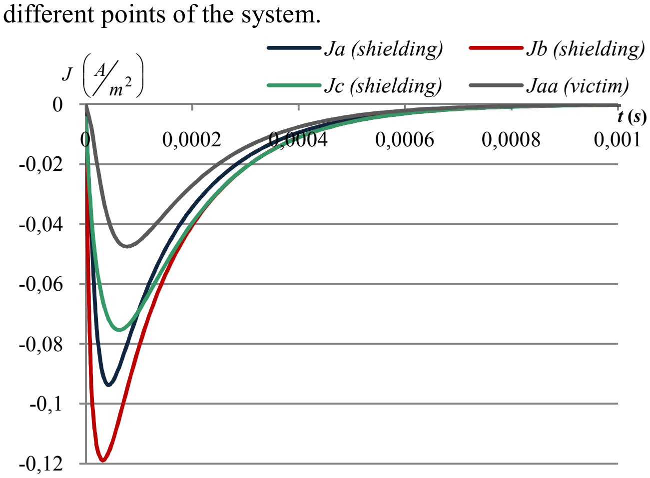

Fig. 2. Simulation results for the current density in some elements of the shielding with $\mu = \mu_o$ and in elements of the victim conductor.

The peak value of the victim current density is about 0.045 (A/m^2) and this can be compared with 0.14 (A/m^2), which was obtained when analyzing the system without the shielding.

Fig. 3 shows some results obtained now considering a magnetic shielding with σ = 5.8·10^2 (S/m) and $\mu = 60 \cdot \mu_o$. In this case, the static FEM tool is indicated to provide the data used to obtain the [C] matrix of the procedure.

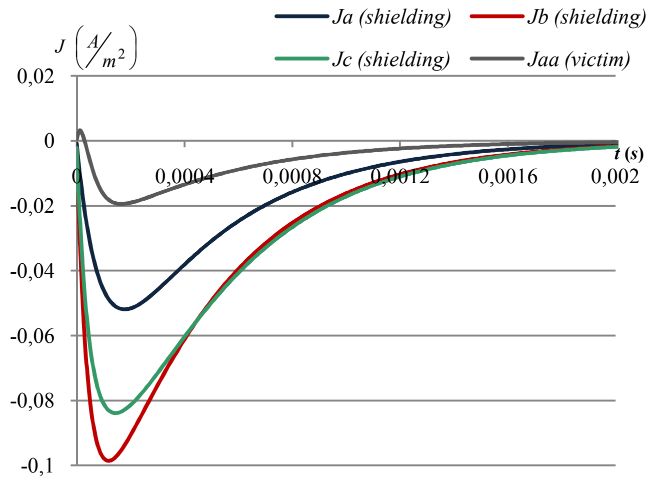

Fig. 3. Simulation results for the current density in some elements of the shielding with $\mu = 60 \cdot \mu_o$ and in elements of the victim conductor.

The results show a more pronounced reduction of the victim current, due to the more effective action of the magnetic shielding. As expected, the response time of the system is slower, as can be seen in the front times and tails of the waveforms.

IV. Conclusions

The paper has presented an efficient numerical procedure for time domain analysis of transient currents. It can be extended to non-homogeneous media through the application of a static FEM tool available in MATLAB.

A particular configuration of source and victim conductors with a shielding box was analyzed and the results have shown the effect of the permeability of the shielding on the victim conductor current, as well as on the current distribution in the shielding body.

V. References

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